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What is Mechanics?

- Kinematics:
  Kinematics is the science which describe motion without any reference to "forces".
- Dynamics:
  Dynamics is the science which describe the effect that "forces" have on motion.
- Kinematics + Dynamics = Mechanics

→ Reference book :
"PHYSICS for scientists and engineers"
Author : LERNER

Lecture 1 : kinematics

Lerner chapter 2,4 and 5

From Greek : « kinema » which mean motion

I. Translation and Rotation

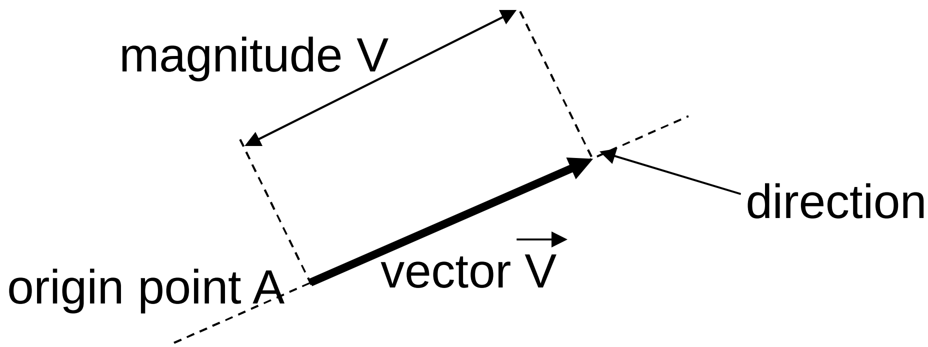
- When you throw an object, all of its parts do not move in exactly the same way
  →rotations (composition of 3 rotations)
  →translations (composition of 3 translations)
- We need to choose a point of this objet to describe its motion.
- Often, we use the center of mass for this object

II. Why do we use the concept of vector ?

- We throw a ball vertically or horizontally. We study the motion of its center of mass : point C
- To describe its motion, we need to its position from a reference point : origin point O.
- The distance between O and C is OC.
  OC is just a number and there is no more information about the motion.
- But it is not sufficient to describe the direction.
  C go to the top or to the bottom, go to left or right
- So we need a new quantity : the vector $\overrightarrow{OC}$ which is an arrow with a magnitude (distance) **and** a direction.

III. SCALAR AND VECTOR ?

- SCALAR :
  A scalar is just a number like distance or mass.
- VECTOR :
  A vector V is a quantity that has both a magnitude (ou V) and a direction.
  Two vectors are EQUAL if and only if they have the same magnitude and same direction.
  But, in physics, we draw the arrow from an origin point.



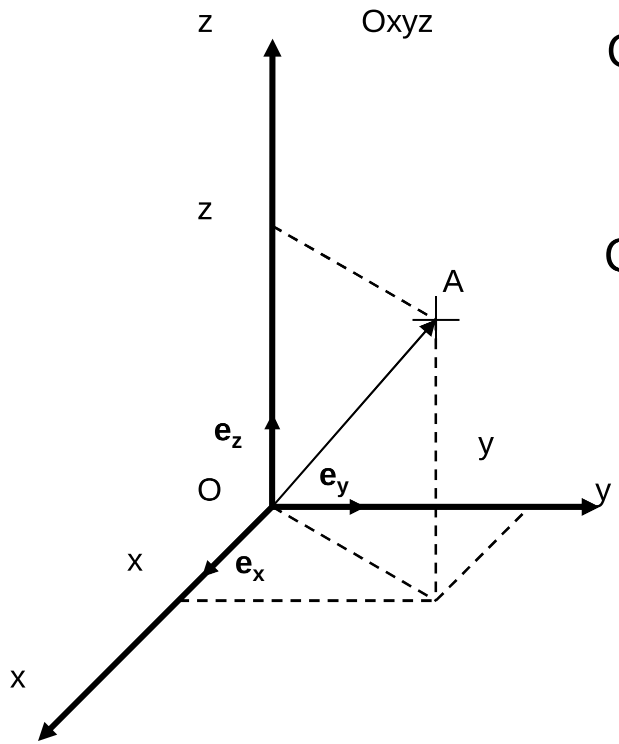
IV. Frame and coordinate system

- Frame : « a point of view about the motion »
  A frame R is a solid of reference which we use to describe a motion.
  Example : A man is seated in a train which have a rectilinear motion. In relation to the train, this man don't move. But in relation to the ground, this man have a retilinear motion. So motion depends on who describe this motion.
  Here we are 2 frames : train frame and ground frame.
  We cannot describe a motion without define a reference frame.
- Coordinate system : « a language to describe the motion »

  In relation to a frame, we need to know position (x,y,z) and the time t for a point if we want to describe its motion.
  To make a coordinate system, we need to define an original point O to define x,y,z and we need to define an original time (time zero) to define the time t of an event.
  We have a lot of coordinate system because if we change the original point all coordinates change.

V. Position and displacement

In a reference frame, we define an origin point O and 3 unit vectors $\left(\overrightarrow{e_x}, \overrightarrow{e_y}, \overrightarrow{e_z}\right)$

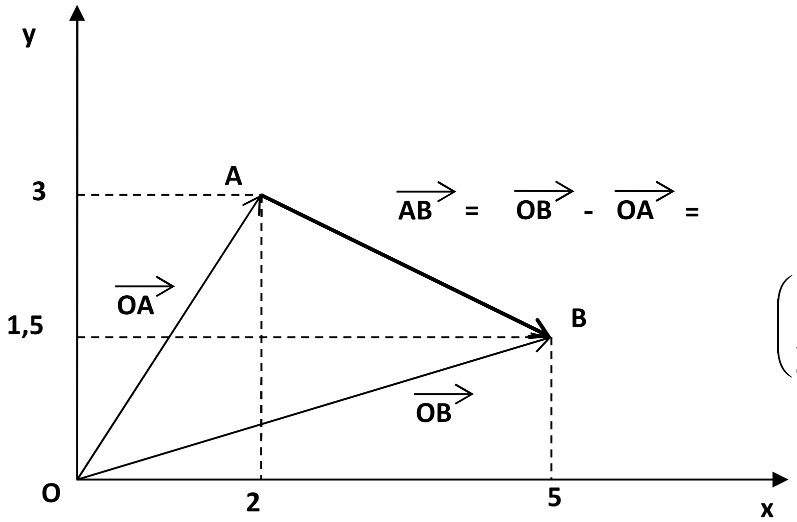


Coordinates of a point A are : $A \left| \begin{matrix} x \\ y \\ z \end{matrix} \right._{Oxyz}$

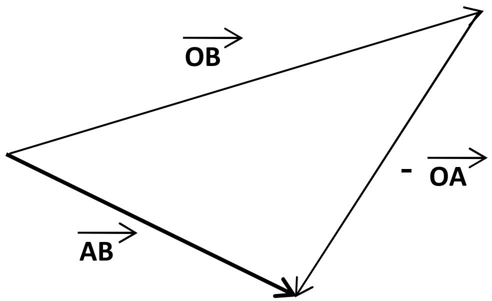
Components of vector position $\overrightarrow{OA}$ are :

$$\overrightarrow{OA} = x\,\overrightarrow{e_x} + y\,\overrightarrow{e_y} + z\,\overrightarrow{e_z} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{Oxyz}$$

To describe motion, we need to specify location of the object at all times. Displacement vector is a vector quantity that points from an objects initial position to its final position and has a magnitude that equals the shortest distance between the two points.



$$\begin{pmatrix} x_B - x_A = 5 - 2 = 3m \\ y_B - y_A = 1{,}5 - 3 = -1{,}5m \end{pmatrix}$$



V. Velocity

In relation to a frame R, velocity is a vector which represent the change in position over a time interval.

- Trajectory : In relation to a frame, trajectory of a point is the path followed by this point.
  When we throw a ball, its trajectory is a parabola.
- A point A follow its trajectory over a time interval : $\Delta t = t_2 - t_1$. Displacement vector is $\overrightarrow{A_1A_2} = \overrightarrow{OA_2} - \overrightarrow{OA_1}$
- Mean velocity : $\overrightarrow{V}_{A/R\ moy} = \dfrac{\overrightarrow{A_1A_2}}{t_2 - t_1}$
- Instaneous velocity : If Δt is infinitesimally close to zero,

$$\overrightarrow{V}_{A/R} = \lim_{t2-t1 \to 0} \frac{\overrightarrow{A_1A_2}}{t_2 - t_1} = \lim_{t2-t1 \to 0} \frac{\overrightarrow{OA_2} - \overrightarrow{OA_1}}{t_2 - t_1} = \frac{d\overrightarrow{OA}}{dt}$$

The instantaneous velocity of point A is the derivative with respect to the time of the vector position.

Instantaneous velocity in A is always tangent with the trajectory in this point.

VI. Acceleration

In relation to a frame R, acceleration is a vector which represent the change in velocity over a time interval.

- Change in velocity :

$\overrightarrow{V_{A1/R}}$ Velocity at time t_1 $\overrightarrow{V_{A2/R}}$ Velocity at time t_2

$\overrightarrow{V_{A2/R}} - \overrightarrow{V_{A1/R}}$ is the change in velocity of A between t_1 and t_2

It is noticed that this vector is directed towards “the interior” of the trajectory.

- Acceleration : In relation to a frame R, if time interval $\Delta t = t_2 - t_1$ is infinitesimally close to zero,

$$\overrightarrow{a_{A/R}} = \lim_{t2-t1 \to 0} \frac{\overrightarrow{V_{A2/R}} - \overrightarrow{V_{A1/R}}}{t_2 - t_1} = \frac{d\overrightarrow{V_{A/R}}}{dt}$$

Acceleration of point A is the derivative with respect to the time of the vector velocity.

Exercise 1

At the begining, a car is motionless. The car accelerates uniformly during 10 s with an acceleration of 3 m/s². The motion is rectilinear.

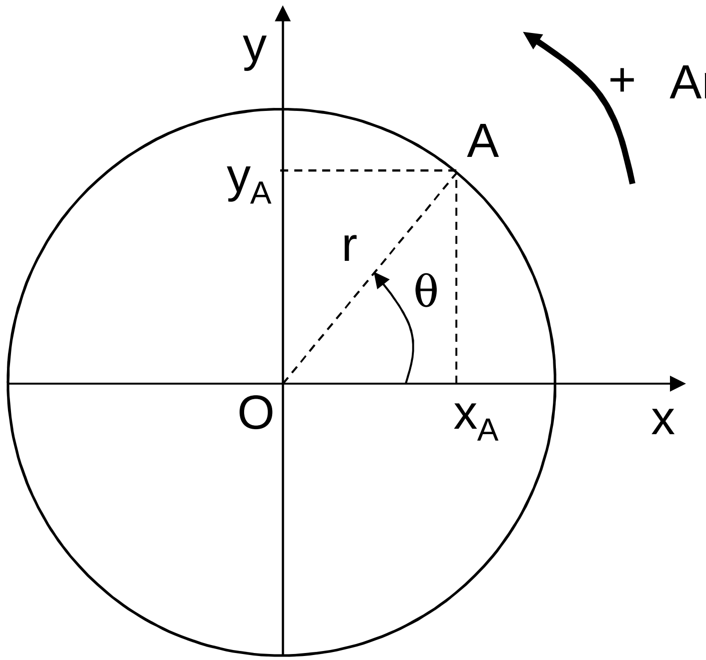
We use a x-axis to locate the position of the center of mass C.

1. Plot acceleration a_x versus time.
2. Plot velocity v_x versus time.
3. Plot position x versus time.

VII. Circular motion (Lerner chapter 5)

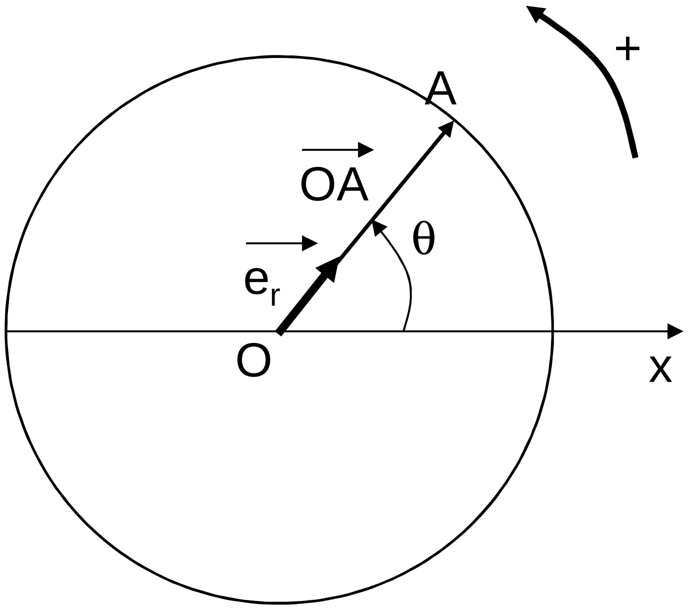
To describe motion of a point, it is sometimes easier to have a mobile coordinate system which follows the motion.
In the continuation we work in a reference frame R and we have a circular motion of point A.

- Polar coordinates (r,θ)



$r = OA$
$x_A = r\cos(\theta)$
$y_A = r\sin(\theta)$

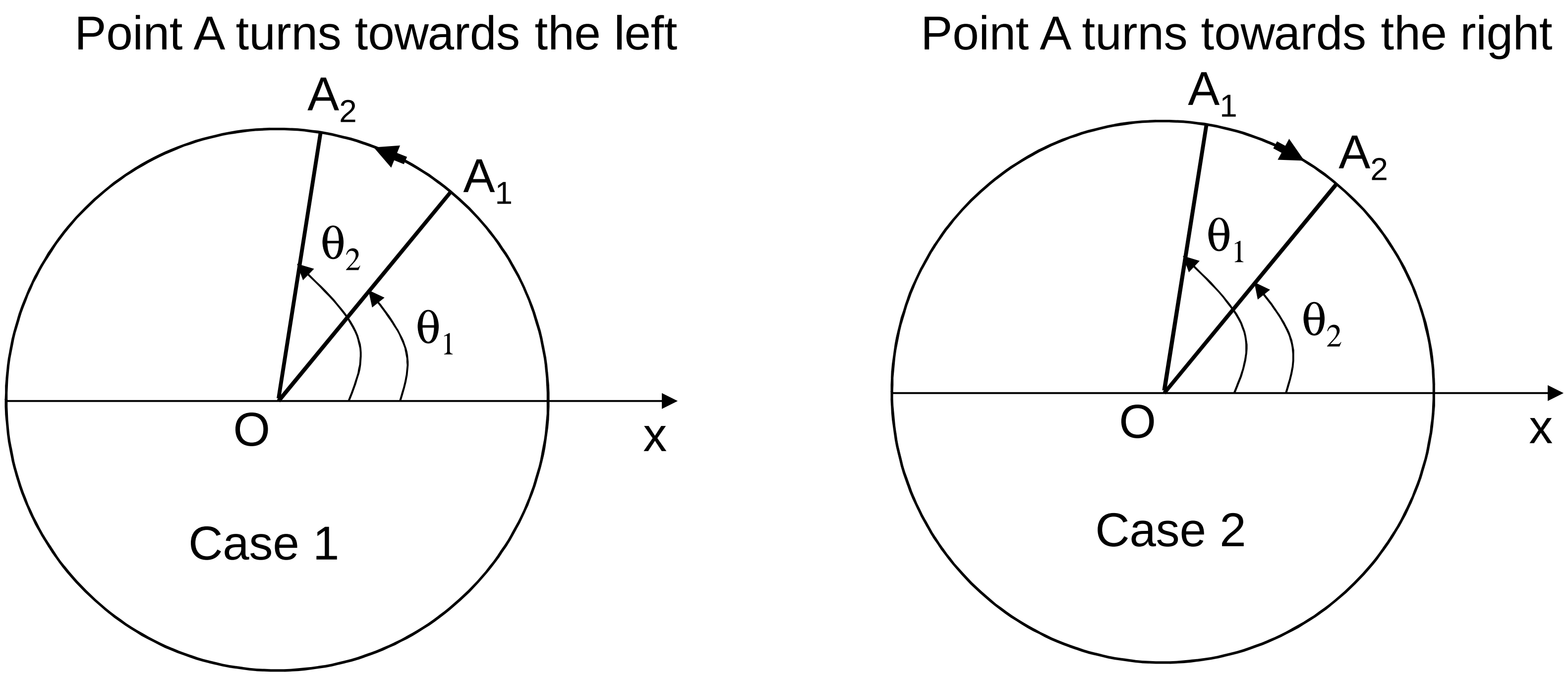
- Polar components of position vector



Radius vector $\vec{e_r} = \frac{\vec{OA}}{OA} = \frac{\vec{OA}}{r}$

$$\vec{OA} = r\,\vec{e_r}$$

Angular velocity

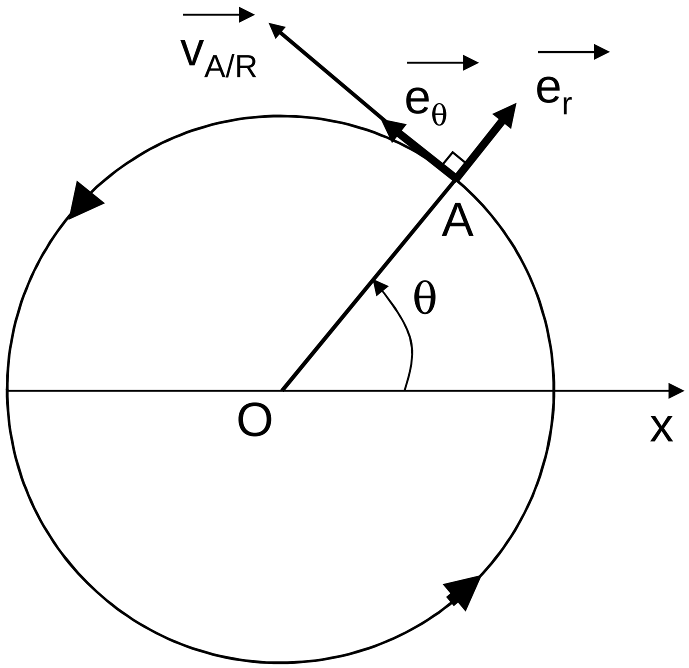


$$\omega = \lim_{t_2 - t_1 \to 0} \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{d\theta}{dt} (rad/s)$$

$$\Rightarrow \begin{cases} \omega > 0 \text{ in the case } 1 \\ \omega < 0 \text{ in the case } 2 \end{cases}$$

- Polar components of velocity vector $\overrightarrow{v_{A/R}}$

Velocity of point A is always tangent of the circle.

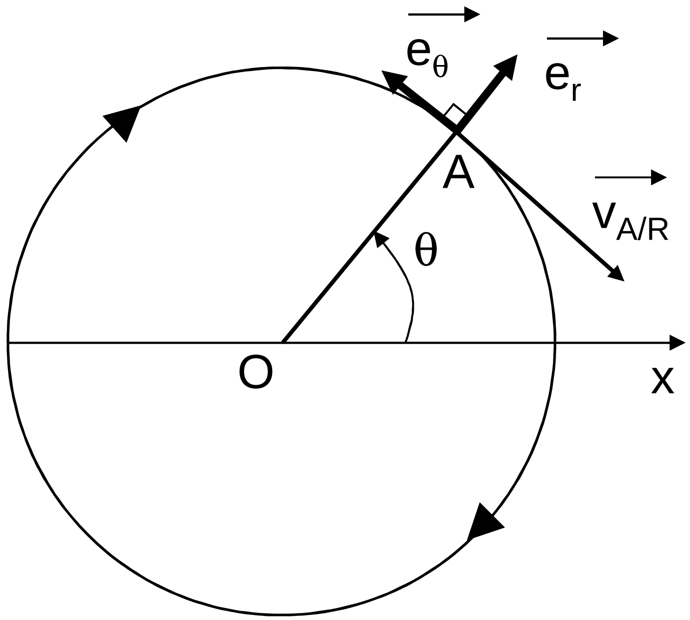


orthoradial vector $\overrightarrow{e_\theta}$

$$\overrightarrow{e_\theta} \perp \overrightarrow{e_r}$$

$$\left\| \overrightarrow{e_\theta} \right\| = 1$$

$$\overrightarrow{v_{A/R}} = r\frac{d\theta}{dt}\overrightarrow{e_\theta} = r\omega\,\overrightarrow{e_\theta} = v\,\overrightarrow{e_\theta} = \begin{pmatrix} v_r = 0 \\ v_\theta = r\omega = v \end{pmatrix}$$



$$\omega = \frac{d\theta}{dt} < 0$$

$$\overrightarrow{v_{A/R}} = r\frac{d\theta}{dt}\,\overrightarrow{e_\theta} = r\omega\,\overrightarrow{e_\theta} = v\,\overrightarrow{e_\theta}$$

Here $\quad v = r\omega < 0$

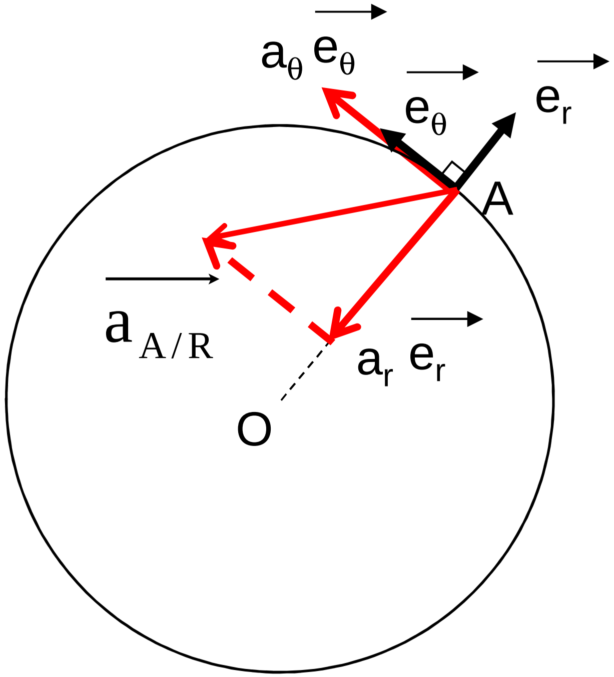
- Polar components of acceleration

$$\overrightarrow{a_{A/R}}(t) = \frac{d\overrightarrow{v_{A/R}}}{dt}$$

However $\overrightarrow{v_{A/R}}(t) = r\,\omega(t)\,\overrightarrow{e_\theta}$

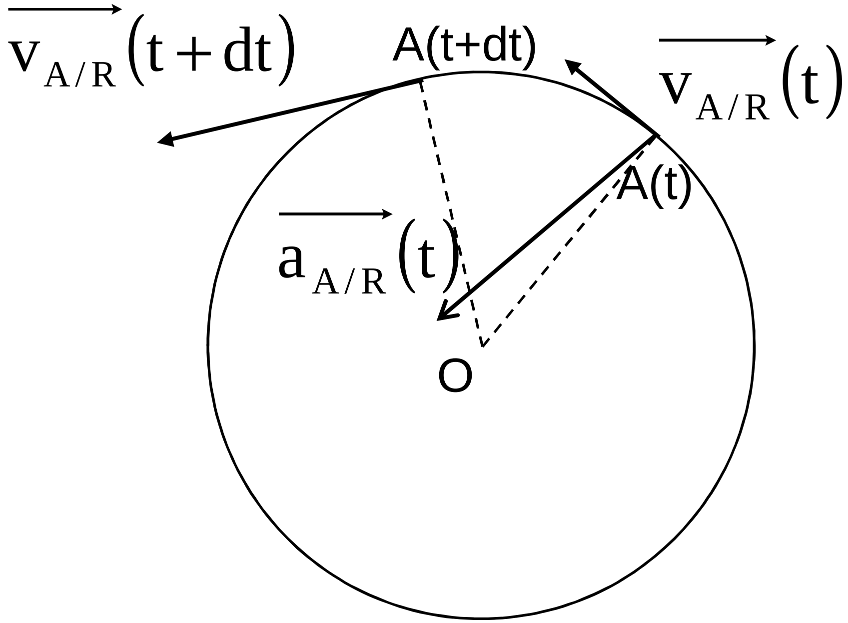
Thus $\overrightarrow{a_{A/R}} = r\frac{d\omega}{dt}\overrightarrow{e_\theta} + r\,\omega(t)\frac{d\overrightarrow{e_\theta}}{dt}$

$$\overrightarrow{a_{A/R}} = r\frac{d\omega}{dt}\overrightarrow{e_\theta} - r\,\omega^2\,\overrightarrow{e_r}$$

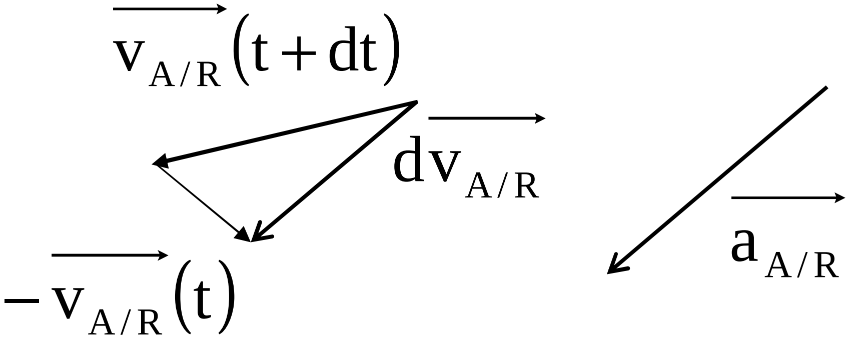


$$\overrightarrow{a_{A/R}} = \begin{pmatrix} a_r = -r\omega^2 = -\dfrac{v^2}{r} < 0 \ \text{(always)} \\ a_\theta = r\dfrac{d\omega}{dt} = \dfrac{dv}{dt}\begin{cases} > 0 & \text{accelerated motion} \\ = 0 & \text{uniform motion} \\ < 0 & \text{decelerated motion} \end{cases} \end{pmatrix}$$

Accelerated circular motion

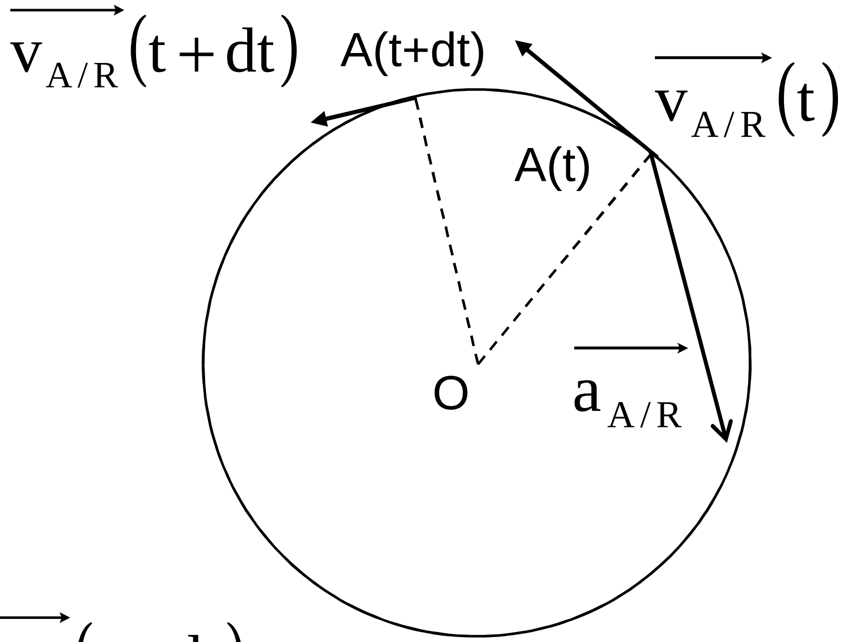


$$\overrightarrow{a_{A/R}} \cdot \overrightarrow{v_{A/R}} > 0$$

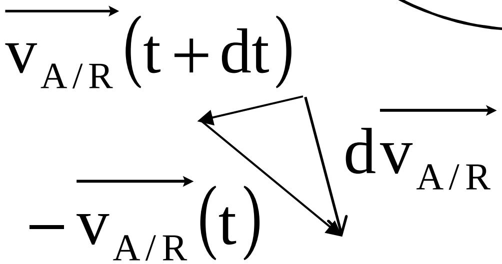


$$\overrightarrow{a_{A/R}}(t) = \frac{\overrightarrow{dv_{A/R}}}{dt}$$

Decelerated circular motion.

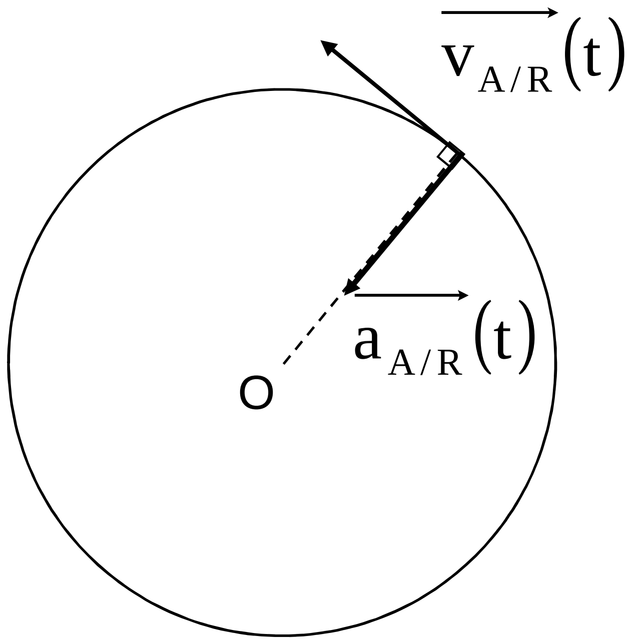


$$\overrightarrow{a_{A/R}} \cdot \overrightarrow{v_{A/R}} < 0$$



$$\overrightarrow{a_{A/R}} = \frac{d\overrightarrow{v_{A/R}}}{dt}$$

Uniform circular motion



$$\overrightarrow{a_{A/R}} \perp \overrightarrow{v_{A/R}}$$

$$\overrightarrow{a_{A/R}} \cdot \overrightarrow{v_{A/R}} = 0$$

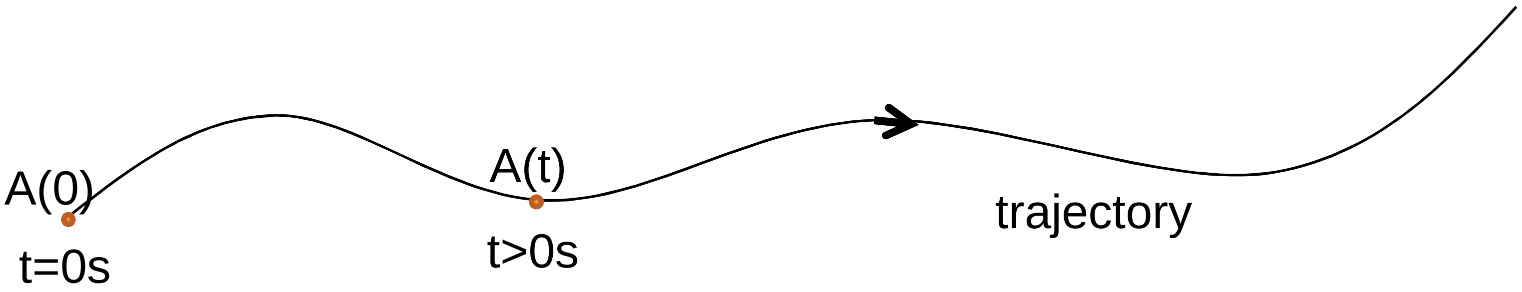
Exercise 2

An astronaut undergoes tests in a centrifugal machine of radius R = 10,0 m. During the first phase, it has a circular motion uniformly accelerated during 10,0 s. During the second phase the movement is circular uniform with an angular velocity equal to 3,00 rad/s during 5,0s. Then a circular motion uniformly decelerated during 5,0 s.

1. Plot angular velocity versus time of this motion.
2. Plot angular position versus time of this motion.
3. Calculate the tangential component and the radial component of acceleration in each phase.

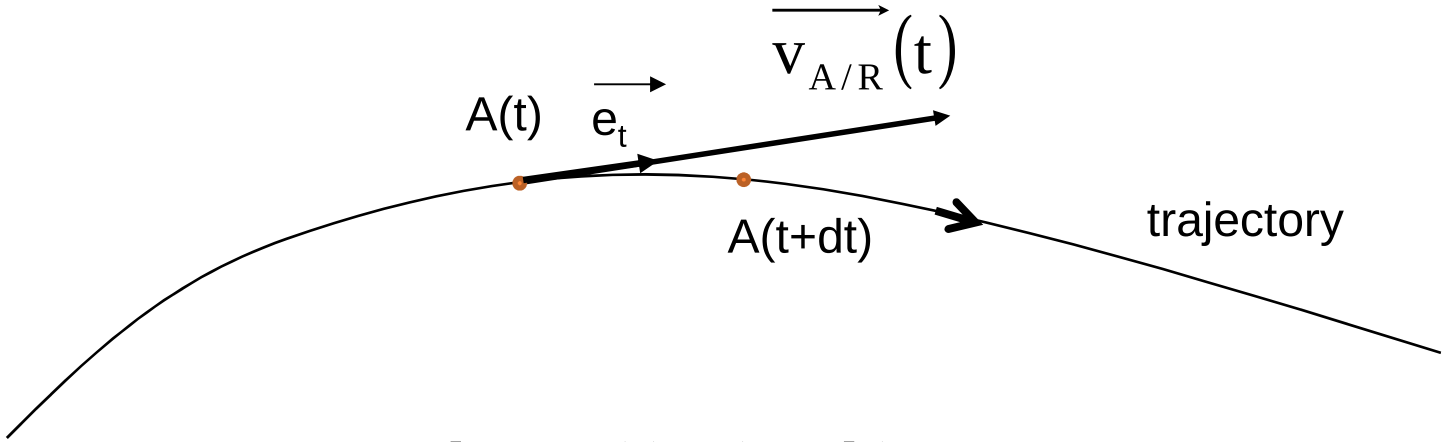
VIII. curvilinear motion (Lerner chapter 5)

- Curvilinear coordinate : s(t)



s(t) = Distance covered between A(0) and A(t)

- Velocity

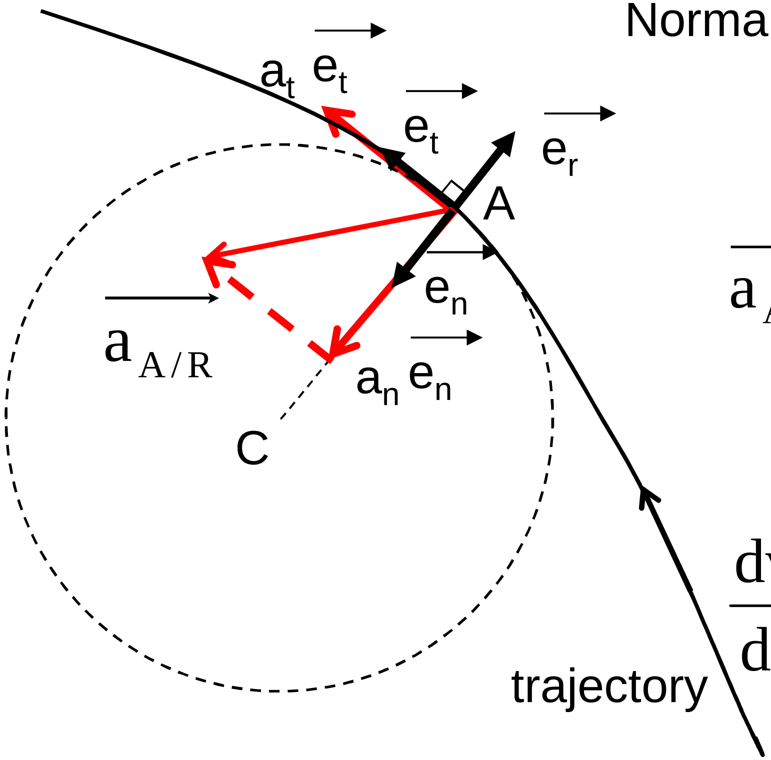


$$ds = A(t)A(t+dt)$$

$$\overrightarrow{e_t} : \text{tangent vector}$$

$$\overrightarrow{v_{A/R}}(t) = \frac{\overrightarrow{A(t)A(t+dt)}}{dt} \approx \frac{ds}{dt}\overrightarrow{e_t} = v\,\overrightarrow{e_t}$$

- Acceleration



Normal vector : $\vec{e_n} = -\vec{e_r} \perp \vec{e_t}$

$$\vec{a_{A/R}} = a_t \vec{e_t} + a_n \vec{e_n} = \begin{pmatrix} a_t = a_\theta = \dfrac{dv}{dt} \\ a_n = -a_r = \dfrac{v^2}{R} > 0 \end{pmatrix}$$

$$\frac{dv}{dt} = \vec{a_{A/R}} \cdot \vec{e_t} \Rightarrow \begin{cases} > 0 \ \text{accelerated motion} \\ = 0 \ \text{uniform motion} \\ < 0 \ \text{decelerated motion} \end{cases}$$

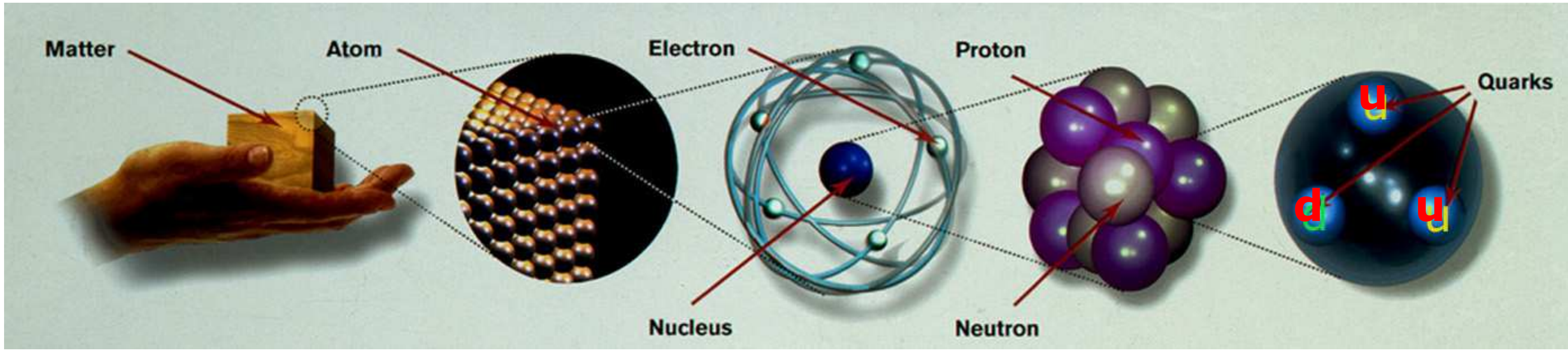
The circle coincides with the trajectory at this moment.
C is the center of the circle.
R=CA is the radius of the circle.

Lecture 2 : Dynamics

Lerner chapter 3, 4, 5, 14, 23

Dynamics studies the cause of the variation of the motion of a body.

I.Constituants of matter



There are only four fundamental forces which explain the interactions between these components.

II. Fondamentals forces

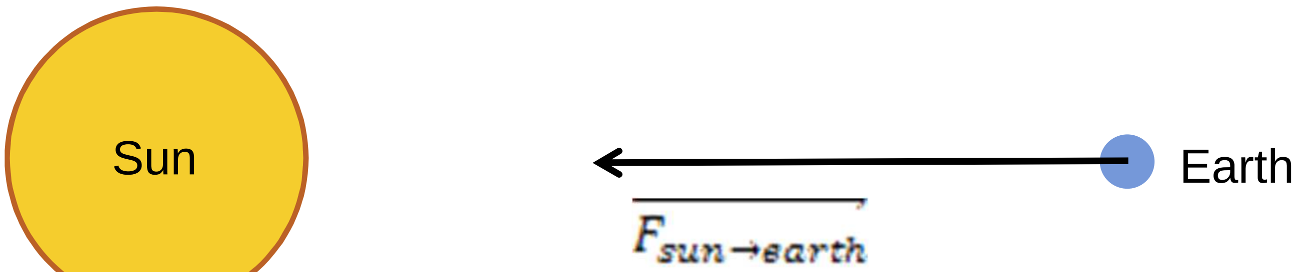
Forces

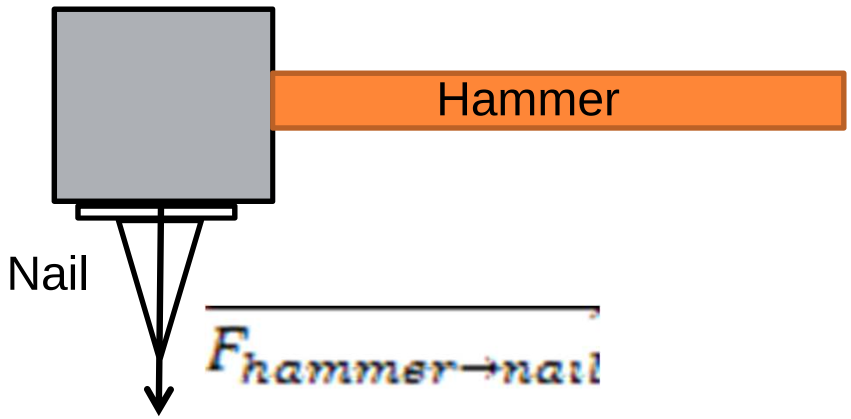
Electro-magnetic	Weak	Strong	Gravity
atoms molecules optics electronics telecom.	beta decay solar fusion	nuclei particles	falling objects planet orbits stars galaxies
inverse square law	**short range**	**short range**	**inverse square law**

III. Vector force

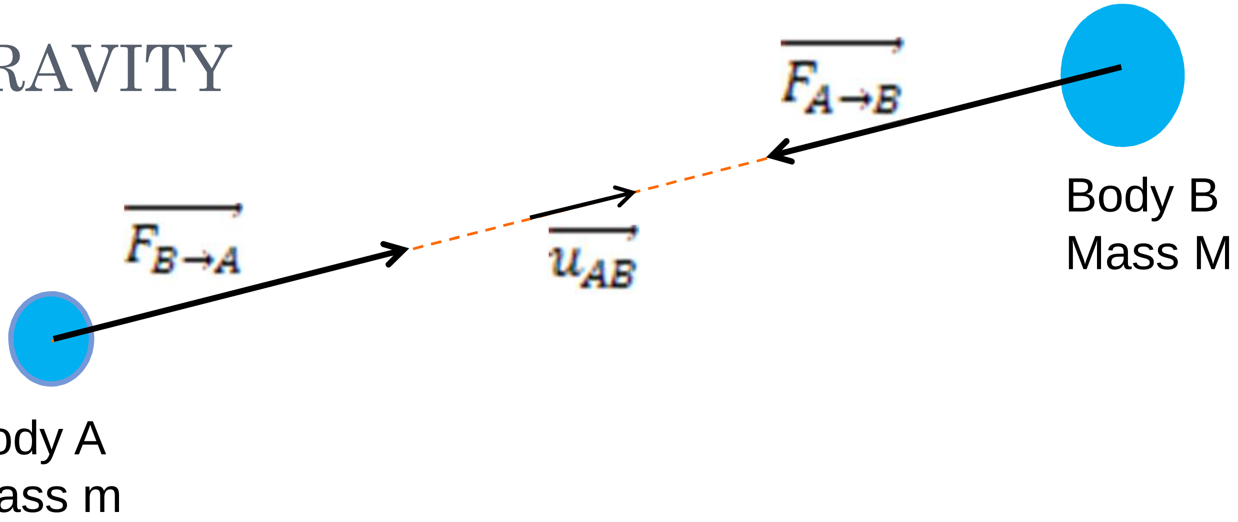
We represent the interactions between the bodies by vectors forces because we need 3 informations about the interaction on a body :

- point of application
- direction
- magnitude (unit : newton (N))





Gravity



Two bodies of mass m and M attract each other with a force proportional to each mass, and inversely proportional to the square of the distance which separates them. This force has as a direction the line passing by the centre of gravity of these two bodies.

A exerts on B the force :

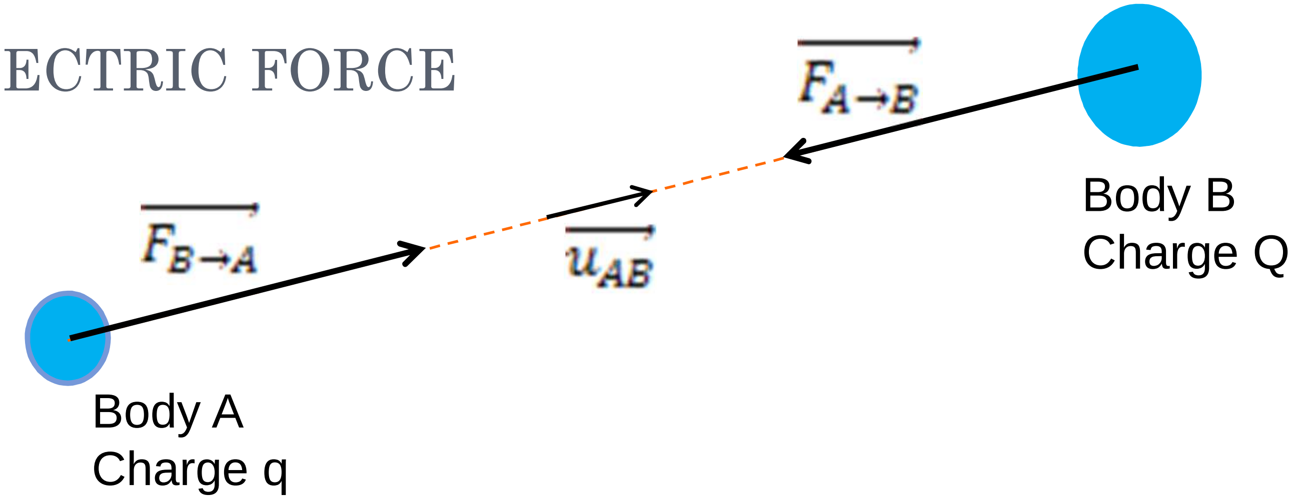
$$\overrightarrow{F_{A\to B}} = -G\frac{m\,M}{AB^2}\frac{\overrightarrow{AB}}{AB} = -G\frac{m\,M}{AB^2}\overrightarrow{u_{AB}}$$

B exerts on A the force :

$$\overrightarrow{F_{B\to A}} = -\overrightarrow{F_{A\to B}}$$

Gravitational constant : G = 6,672 59 . 10^{-11} N.m^2.kg^{-2}

ELECTRIC FORCE



If q and Q are of opposite sign (+- or -+), A and B attract each other.
If q and Q are of the same sign (++ or --), A and B are pushed back.
The electric force is proportional to each charge, and inversely proportional to the square of the distance which separates them. This force has as a direction the line passing by the center of these two bodies.

A exerts on B the force :

$$\overrightarrow{F_{A\to B}} = \frac{1}{4\pi\epsilon_0}\frac{q\,Q}{AB^2}\overrightarrow{u_{AB}}$$

B exerts on A the force :

$$\overrightarrow{F_{B\to A}} = -\overrightarrow{F_{A\to B}}$$

Permittivity of free space : ε_0=8,85.10^{-12} C^2.N.m^{-2}

IV. Force and acceleration

- Important idea of kinematics :
  If the motion of body is modified, then the vector velocity, which is always tangent with the trajectory, was modified. Thus this body has undergoes an acceleration.

- Important idea of dynamics :
  If a body changes its motion, then the body undergoes action of external forces whose sum is proportional to acceleration which modified velocity

V. How to choose the good reference frame to describe the motion?

- We are in a train which runs in straight line with a constant speed.
- We look at a car which runs with a constant speed on a road parallel with the railway.
- Suddenly the train turns on the left. We will have the impression that the car turns on the right. Thus we will suppose that the car underwent a force which made deviate its trajectory.
- But, we know that it is false impression. We turned left.
- Thus the reference frame train is a good reference frame at the beginning of the motion but it becomes a bad reference frame at the end of the motion.

➡ The good reference frames, to describe the forces which explain a motion, are not accelerated.
One calls these particular reference frames :
<u>inertial frames of reference or inertial frames for short</u>.

VI. Inertial frame (Newton's first law)

An inertial frame is a reference frame where any free particle, on which the sum of the external forces is null, is motionless or is animated of a uniform rectilinear motion.

- ⇒ If the sum of the external forces is null, and if the body has an accelerated motion, then the reference frame is not an inertial frame.
- ⇒ In an inertial reference frame, if a body has an accelerated motion then the sum of the external forces is not null.

All inertial frames are equivalent to describe the fundamental laws of physics.
These laws take the same form (equation) in all these frames.

VII. Newton's second Law

In an inertial frame R, the sum of the external forces exerted on a particle P (mass m) is equal to the product of its mass and its acceleration.

$$\sum \vec{F}_{ext \to P} = m \, \vec{a}_{P/R}$$

VIII. Newton's third law

To every force there is always opposed an equal force :
or, the mutual action of two bodies upon each other
are always equal in magnitude and are in opposite directions.

Example : A man draws on a rope fixed on a wall.